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Wormholes in AdS

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Abstract: We construct a few euclidean supergravity solutions with multiple boundaries. We consider examples where the corresponding boundary field theory is well defined on each boundary. We point out that these configurations are puzzling from the AdS/CFT point of view. A proper understanding of the AdS/CFT dictionary for these cases might yield some information about the physics of closed universes.

Keywords: Superstrings and Heterotic Strings, D-branes, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

The general statement of the AdS/CFT correspondence is thought to be that the sum over all geometries with fixed boundary conditions is the same as the partition function of a (conformal) field theory living on the boundary [1, 2, 3].

A puzzle arises if we have euclidean geometries that have more than one disconnected boundary and are connected through the bulk.[1] In this paper we explore this puzzle. Our main results are the construction and analysis of a variety of examples of euclidean geometries with two boundaries. We have found examples where the field theory at each of the boundaries seems perfectly well defined. The theorem in [5, 6], shows that one cannot find examples with positive boundary curvature which solve Einstein's equations with a negative cosmological constant. We found examples once we turn on additional Yang-Mills fields.

The configurations we describe are similar to wormholes in the sense that they connect two well understood asymptotic regions. Our geometries are solutions of the ten or eleven dimensional supergravity actions that arise in string theory. In previous examples of wormhole-like solutions either the two asymptotic regions were not as well understood or they were not solutions of the ten or eleven dimensional supergravity actions [7]–[14]

In section 2 we discuss some general properties of the solutions we have found. In section 3 we discuss the simplest examples based on quotients of hyperbolic space. In section 4 we discuss an example closely related to the standard $AdS_4 \times S^7$ solution, where the boundary field theory is twisted by the fields of a meron. In section 5 we discuss an example closely related to the $AdS_5 \times S^5$ solution, where the boundary theory is twisted by an instanton. In section 6 we point out some of the puzzles raised by these solutions and we speculate on their possible resolution.

2. Generalities

In this paper we consider only euclidean solutions. We will analyze solutions which are asymptotically AdS. We study solutions that have two disconnected boundaries, which are connected through the interior. In this sense they are similar to the euclidean wormholes considered in [7] which connect two asymptotically flat regions. The typical form of the metric for these solutions is

$$ds^2 = d\rho^2 + w(\rho)^2 ds^2_{\Sigma_d} \tag{2.1}$$

where Σ is a compact surface and $w(\rho) \sim e^{|\rho|}$ as $\rho \to \pm\infty$. The two disconnected boundaries are at $\rho = \pm\infty$. Most solutions we consider are reflection symmetric around $\rho = 0$. These solutions have the interesting property that they can be analytically continued into lorentzian signature by replacing $\rho = it$. This analytic continuation describes closed universe cosmologies with spatial surfaces given by Σ which expand from a big bang and collapse to a big crunch, see figure 1.

Our main motivation was to find the field theory interpretation of these solutions. In particular, we wanted to find the field theory interpretation of the closed cosmologies. We could not find a definitive answer for these questions and we offer some speculations at the end of the paper.

[1] Lorentzian geometries with multiple asymptotic boundaries are such that boundaries are separated by horizons [4], as long as the boundaries have more than one dimension. In this case one can interpret the geometries as dual to entangled states of the various field theories living on each boundary.

In most of this paper we focus on constructing various solutions that have the general form (2.1). Before we describe each particular case we would like to describe various general aspects of the solutions. Since the solutions are asymptotically AdS our first goal will be to understand whether the boundary conditions are stable. For example, we want to make sure that we cannot decrease the action by creating a brane in the interior and moving it all the way to the boundary.

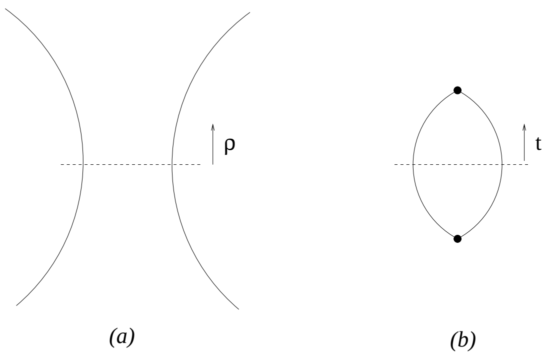

Figure 1: In (a) we sketch a euclidean geometry with two boundaries and with a reflection symmetry around $\rho = 0$. In (b) we sketch the lorentzian geometry that results from the analytic continuation $\rho = it$. It describes a big bang to big crunch cosmology.

These are asymptotic instabilities which arise near each boundary independently and do not come from the fact that the solution has two boundaries. They are related to the fact that the corresponding boundary field theories are not well defined, the boundary theory would not have an action that is bounded below. Examples of these instabilities were discussed in [15], where an instability of this type was found when the boundary has negative curvature. In the field theory they arise due to the conformal coupling of the scalar fields — negative curvature translates into an effective negative mass for the scalar fields. One can go around this problem in two ways, one is to consider positive curvature boundaries. The second is to consider a two dimensional field theory which can be defined on an arbitrary Riemann surface.

Once we consider the two boundary solution it can happen that the bulk euclidean action has some negative modes. This would mean that the solution is a saddle point but not a local minimum. Of course, the euclidean gravity action is not positive definite due to the conformal factor of the metric. This is a problem even in usual euclidean AdS space. Here we will use the prescription in [16], which consists in analytically continuing the integral over the conformal factor to imaginary values. If the solution is asymptotically stable, then any possible negative mode will be localized in the interior, in the region connecting the two boundaries. Some of the solutions we found have negative modes and some do not.

In most of the examples we found, it is possible to find other configurations which have the same asymptotic boundary conditions but consist of two disconnected spaces attached to each of the two boundaries (see figure 2c). The simplest such example is to add an end of the world brane at $\rho = 0$. This end of the world brane could arise from an orbifold or orientifold involving the ρ direction and any number of internal dimensions. But the disconnected geometries could also be a bit more complicated. We will present some examples below.

Beyond perturbative instabilities, we could ask if there is another configuration with lower action. In some cases we find that there is. Then the wormhole solution is, at best, local minimum but not global minimum.

One can wonder if there is a supersymmetric solution with two boundaries. We did not find any simple example. There is a simple argument that rules out solutions of the form (2.1), with a reflection symmetry at $\rho = 0$ which analytically continues to a time reflection symmetry. These solutions cannot be supersymmetric since lorentzian supersymmetric solutions must have a timelike or null killing vector [17]. But it is clear that the analytic continuation of (2.1) cannot have such killing vectors if the spatial sections are compact.[2]

Finally we should point out a difference between the wormholes we find and the axionic wormholes discussed in [7]. An axionic wormhole in AdS is such that the two wormhole ends can be moved around AdS by the AdS isometries. In contrast, in our solutions some of the AdS isometries are broken by the boundary conditions in such a way that the wormhole ends are localized in the central region of the geometry, which would correspond to the IR of the boundary theory.

3. Quotients of hyperbolic space

The simplest example is constructed as follows [18]. Let us start with the euclidean AdS (or hyperbolic space) metric written in terms of hyperbolic slices

$$ds^2_{H_{d+1}} = d\rho^2 + \cosh^2 \rho ds^2_{H_d} \tag{3.1}$$

where $\rho \in (-\infty, \infty)$.

Even though the coordinates in (3.1) seems suggestive of two boundaries at $\rho = \pm\infty$, these spaces have only one boundary, which is a sphere S^d. This can be checked by a conformal transformation or by following the coordinate transformation between (3.1) and the sphere slicing. More explicitly, each boundary would be a hyperbolic disk H_d. However the two disks are joined at their boundaries. In fact, we can think of each disk as half of the sphere S^d. In other words, when we consider a quantum field theory in hyperbolic space H_d, we need some boundary conditions at the boundary of H_d. In this case the boundary conditions are the following. We take two disks and put "transparent" boundary conditions at the boundary, i.e. the boundary conditions we have when we consider the sphere and we separate the sphere into two hemispheres. This situation was studied in [19]. Of course, we could consider the field theory in hyperbolic space H_d with other boundary conditions (i.e. non-transparent boundary conditions). We expect that this will introduce an end of the world brane at $\rho = 0$. The precise form of this end of the world brane will depend on the boundary conditions. For example, we can consider the M2 brane field theory and put the boundary conditions that result when an M2 brane is ending on the M-theory end of the world brane. The supergravity solution is (3.1) with an end of the world brane at $\rho = 0$ so that we have only one boundary. In general, this end of the world brane can be an orbifold or an orientifold which reflects the ρ coordinate and any number of internal coordinates. This orbifold or orientifold should which should be consistent with the RR fluxes present in the system.

[2] If the spatial sections are non-compact then one can have supersymmetric solutions of this form. An example is AdS space written in the hyperbolic slicing.

JHEP02(2004)053

We can now make a quotient of hyperbolic slices H_d in (3.1) by a discrete subgroup of the hyperbolic symmetry group, $\mathrm{SO}(1,d)$. We can pick this group Γ so that $\Sigma_d = H_d/\Gamma$ is a compact, smooth, finite volume surface. Now the resulting space has a metric of the form (2.1) with two disconnected boundaries.[3] In the case $d=2$, Σ_2 is a constant curvature Riemann surface of genus $g \geq 2$.

The lorentzian continuation of (3.1) gives a big bang/big crunch cosmology considered in [20].[4]

3.1 Perturbative analysis

Here we consider potential perturbative instabilities. We consider a metric of the form

$$ds^2 = d\rho^2 + \cosh^2\rho\, ds^2_{\Sigma_d} \tag{3.2}$$

where $\Sigma_d = H_d/\Gamma$ is a constant negative curvature compact manifold. Let us consider scalar fields. We will be interested in eigenfunctions $(-\nabla^2 + m^2)\phi = \lambda\phi$ with $\lambda \leq 0$ which are normalizable. The lowest values of λ will be achieved for functions that are independent of the coordinates on Σ_d. After the change of variables $(1-w) = \cosh^2\rho$ the eigenvalue equation becomes a hypergeometric equation with parameters

$$\alpha = \frac{d}{4} + \sqrt{\left(\frac{d}{4}\right)^2 + \frac{\tilde{m}^2}{4}}\,, \qquad \beta = \frac{d}{4} - \sqrt{\left(\frac{d}{4}\right)^2 + \frac{\tilde{m}^2}{4}}\,, \qquad \gamma = \frac{1}{2} \tag{3.3}$$

where $\tilde{m}^2 = m^2 - \lambda$. The two solutions are $F(\alpha, \beta, \frac{1}{2}; w)$ and $w^{1/2}F(\frac{1}{2}+\alpha, \frac{1}{2}+\beta, \frac{3}{2}; w)$. Both are regular at $\rho = 0$ in terms of the ρ coordinate. Demanding that the solution is normalizable at infinity we find the condition

$$\sqrt{\left(\frac{d}{2}\right)^2 + \tilde{m}^2} = \frac{d}{2} - n \tag{3.4}$$

where $\frac{d}{2} \geq n > 0$ is an integer. We can rewrite this as

$$\lambda = \left(\Delta - \frac{d}{2}\right)^2 - \left(\frac{d}{2} - n\right)^2; \qquad \Delta = \frac{d}{2} + \sqrt{\left(\frac{d}{2}\right)^2 + m^2}\,. \tag{3.5}$$

We see that we should worry about possible negative modes only for relevant operators.[5]

The reader might be puzzled by the following. We know that in AdS we do not have any negative modes, then: why do negative modes arise when we write AdS in hyperbolic

[3] The boundaries are connected through the bulk, but they are disconnected through the boundary.

[4] [20] proposed a description of these cosmologies in terms of a quotient of the lorentzian boundary theory. It is not clear how to construct these theories. Here we are suggesting that the cosmological solutions are somehow related to the well defined boundary euclidean theories. However, we did not specify the precise relation.

[5] Negative mass square fields can sometimes have more subtle boundary conditions which lead to an expression for the conformal weight involving the other branch of the square root in (3.5) [21]. In these cases the first expression in (3.5) is still valid (but there is a minus sign in the second).

JHEP02(2004)053

coordinates? The point is that these negative modes are present only after we quotient the constant ρ slices. If these slices were non-compact hyperbolic spaces, as in (3.1) then there would be no negative modes since the wavefunctions need to be normalizable on these slices and this forces them to vary along these constant ρ slices.[6]

Let us consider some examples. Let us set d=2 so that we are considering AdS_3/CFT_2. In particular, we consider the example of type-IIB string theory on $AdS_3 \times S^3 \times K3$. This theory has fields that correspond to operators with conformal weights $\Delta = L_0 + \bar{L}_0 = 1, 2, \ldots$ In this case, there are no negative modes but a zero mode appears for an operator of $\Delta = 1$. This operator has spins $(1/2, 1/2)$ under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ rotations of S^3. We can project it out if we quotient the theory by a $Z_N \subset \mathrm{U}(1)_L \subset \mathrm{SU}(2)_L$. This quotient does not lead to fixed points and removes the potentially problematic field. The resulting conformal field theory at the boundary has (4,0) supersymmetry and was studied in [23].

As another example, set $d = 4$, so that we have AdS_5/CFT_4, then $2 \leq \Delta < 3$ leads to negative modes and $\Delta = 3$ leads to a zero mode. In the case of $AdS_5 \times S_5$ we have operators with $\Delta = 2, 3$ in the spectrum. In this case we can analyze in a bit more detail the action of one of the zero modes associated to operators with $\Delta = 3$. We choose the operator that gives an equal mass to three of the four Yang Mills fermions. Then the classical action for the corresponding field has the form [24]

$$L \sim \frac{1}{2}(\partial\phi)^2 - \frac{3}{8}\left[3 + \left(\cosh\frac{2\phi}{\sqrt{3}}\right)^2 + 4\cosh\frac{2\phi}{\sqrt{3}}\right]. \tag{3.6}$$

This lagrangian shows that the quartic order term in ϕ is negative, so that what started out at as a zero mode in the classical approximation ends up as a negative mode.

In this example one could imagine projecting out all operators with $\Delta < 3$ by performing a quotient on the fivesphere, $AdS_5 \times (S^5/\Gamma)$. If Γ contains elements that have fixed points on S^5 then we have to worry about the extra fields that can live at these fixed points. On the other hand, it is possible to see that there is no subgroup of SO(6) that does not give fixed points and, at the same time, projects out all the $\Delta = 2$ operators, which are in the **20** of SO(6).[7]

Finally note that a quotient of euclidean version of the solution [27] gives us a configuration where the value of the dilaton in the two asymptotic boundaries is different.

3.2 Non perturbative instabilities

For these manifolds further instabilities arise due to possible brane creation. These are backgrounds with a p-form field strength, so we should worry about the possibility of creating branes that screen, or partially screen, this fieldstrength. This problem was analyzed

[6]There have been a few results in the mathematical literature, concerning the spectrum of the scalar laplacian in complete hyperbolic manifolds, in their quotients and in more general asymptotically hyperbolic Einstein manifolds. It was shown that for the complete hyperbolic manifolds, the eigenvalues are always continuous and greater than $d^2/4$. For quotients of such spaces, and for other asymptotically hyperbolic manifolds, there can be also discrete eigenvalues below $d^2/4$. The exact statements and conditions for their existence can be found in [22].

[7]This can be shown easily for abelian subgroups. For non-abelian subgroups the classification in [25] and [26] shows that all non-abelian subgroups lead to fixed points.

JHEP02(2004)053

in [15]. We now summarize that discussion. We consider a euclidean $(d-1)$-brane wrapping the d dimensional slices of the metric (3.1). We assume that the brane is charged under the same $(d+1)$-form potential whose field strength supports the background. In units where the AdS radius is one its action can be written as

$$S \sim (\text{Area} - d\,\text{Volume}) \sim (\cosh\rho)^d - d\int_0^\rho d\rho'(\cosh\rho')^d \sim -\frac{2d}{2^d(d-2)}e^{(d-2)\rho} + \cdots \quad (3.7)$$

where we extracted the leading dependence for large ρ. We see that for large ρ the leading contribution to (3.7) is negative, so that if we create a brane antibrane pair and we move one of them to $\rho \to \pm\infty$ we can decrease the action. In fact, for large ρ, the position of the brane can be viewed as a conformally coupled scalar from the field theory point of view. Since the field theory is on a negatively curved space this conformal coupling leads to an effective negative mass term for the scalar, which causes an instability. This instability under brane creation is a non-perturbative instability, in the sense that we need to overcome a barrier of order $\frac{1}{N} \sim \frac{1}{g_s}$ in order to create the branes. This discussion holds for situations where we have compactified the constant ρ sections. If we do not compactify them then the negative mass of these scalar fields is not a problem since a conformally coupled field obeys the Breitenlohner-Freedman bound in d dimensions. In fact, for a conformally coupled scalar in hyperbolic space, H^d, we have $R^2m^2_{\text{conf}} - R^2m^2_{BF} = \frac{1}{4}$ where R is the radius of hyperbolic space, m_{conf} is the effective mass that results from the coupling to the constant negative curvature for a conformally coupled scalar and m_{BF} is the Breitenlohner-Freedman bound.

Note that the instability that we are discussing here has nothing to do with the fact that the manifold has two boundaries, it is purely due to the asymptotic geometry near each of the boundaries. It would be present as long as the boundary has negative curvature and the spacetime solves Einstein's equations [15]. So in these cases the CFT is not well defined and we cannot find a minimal action spacetime configuration with these boundary conditions. See [28] for a more general discussion on the boundary manifolds which lead to instabilities of this type.

One would expect that adding a large enough mass term to the lagrangian would remove the instabilities. For example, in $AdS_5 \times S^5$ we can turn on the operator that turns on a mass for three of the fermions. This also induces a mass for the scalars by supersymmetry. When the field theory is in flat space these deformations were considered in [24, 29]. One could repeat their analysis on a negatively curved hyperbolic space. It could be that the fivebranes that appear in flat space [29] will also appear here and disconnect the two asymptotic regions.

3.3 Correlation functions

We can compute the correlation functions of boundary operators using gravity. Before doing the quotient we can compute the bulk to boundary propagator in hyperbolic coordinates and we find

$$G(r,\rho;r_0) \sim \frac{1}{(\cosh\rho)^\Delta\,[\cosh s - \tanh\rho]^\Delta} \quad (3.8)$$

where (r,ρ) label a point in the bulk, r_0 labels a point in the boundary and s is the distance between the points r and r_0 measured with the boundary hyperbolic metric.

Now we can compute the two point function by taking $\rho \to \infty$, renormalizing by a factor of $e^{\Delta\rho}$ to obtain[8]

$$\langle O(r,\theta^i)_1 O(r',\theta'^i)_1\rangle \sim \frac{1}{[\sinh s/2]^{2\Delta}} \tag{3.9}$$

for operators on the same boundary and

$$\langle O(r,\theta^i)_1 O(r',\theta'^i)_2\rangle \sim \frac{1}{[\cosh s/2]^{2\Delta}} \tag{3.10}$$

for operators on opposite boundaries, where s is the distance between the two points measured with the boundary hyperbolic metric. This is the result for non-compact hyperbolic slices. If we want to consider the theory on the quotient H_d/Γ we need to sum over all images.[9] This sum over images might diverge. By estimating the sum as an integral one can see that the sum converges if $\Delta > d-1$. It is interesting that this is the same condition that eliminates zero and negative modes for the field in the interior (see (3.5)). We see that after summing over images the correlator across the two boundaries (3.9) will generically give a non-zero answer with some coordinate dependence.

Note that if the operators in (3.9) carry any charge under a global symmetry then these correlators vanish due to the fact that there is a gauge field in the bulk whose Gauss's law prevents any charge transfer across boundaries. In other words, even though the correlators computed as in (3.10) are nonvanishing the full result vanishes once we integrate over the gauge field in the bulk that is associated to the global symmetry in the boundary theory. The net result is that there is no charge transfer among the two boundaries.

3.4 AdS_3 and the change in moduli of Riemann surfaces

In this subsection we consider the three dimensional case (though similar remarks might apply to higher dimensional cases). If we have a 2d Riemann surface of constant curvature this surface will have moduli t^α which specify its shape. The moduli of the 2d Riemann surface can be different on the two sides. In fact there is a theorem ("the Bers simultaneous uniformization theorem"), which states that the solutions are in one to one correspondence with a pair of points in Teichmuller space [32]. These two points are the values of the moduli of the 2d Riemann surface at the two boundaries. We can represent a Riemann surface as the quotient of H_2 by a fuchsian discrete group $\Gamma \subset \mathrm{SL}(2,\mathbb{R})$. If we quotient H_3 by the same group, Γ, now viewed as a subgroup of $\mathrm{SL}(2,\mathbb{C})$ we end up with a geometry where the moduli of the Riemann surface are the same on the two boundaries. Quotienting H_3 by so called "quasi-fuchsian" groups gives us three dimensional spaces that join Riemann surfaces with different moduli.

In addition, given a Riemann surface on the boundary, it is possible to find a geometry that ends on it and has no other boundary. This is a geometry that results by quotienting

[8]The correct method is a bit more involved, but this gives us the correct result up to a Δ dependent factor [30, 31].

[9]The sum over images gives us the result when we neglect interactions in the interior. Once we take into account interactions the result is not given by the sum over images. Correspondingly, the full gauge theory correlators are not given by a sum over images since the field theory is an interacting field theory.

JHEP02(2004)053

H_3 by a so called Schottky group, see [33] for further discussion.[10] It would be nice to see if the geometry with disconnected boundaries has larger or smaller action than the geometry which connects the two boundaries.[11]

3.5 A rather stable example

Let us consider an AdS_3/CFT_2 example. An ordinary two dimensional conformal field theory can be defined on any Riemann surface. On the other hand we saw above that there is generically an instability under brane creation. In the two dimensional case the computation in (3.7) gives an action going like $S \sim -\rho$, for large ρ. We should note however that the computation in (3.7) assumed the existence of a D-brane in the bulk whose tension, T, is equal to its charge, q, (in some units), so that the leading contribution to (3.7) cancels. In supersymmetric backgrounds we have a BPS bound ensuring that $T \geq q$. However there is no particular reason why there should exist a brane that saturates this equality.

Let us be more concrete. Consider the example of $AdS_3 \times S^3 \times K3$. Let us consider this system with Q_1 units of NS electric flux on AdS_3 and Q_5 units of NS magnetic flux on S^3. If the values of all RR fields on the $K3$ vanish then a fundamental string (or an NS fivebrane) has $T = q$ and leads to an instability as discussed above. In [15] it was observed that the dual conformal field theory is singular since its target space is non-compact. This target space can be thought of as the moduli space of Q_1 instantons of SU(Q_5) gauge theory on $K3$. This non-compact region comes from small instanton singularities. If we turn on some particular RR fields on $K3$ then the instantons become non-commutative instantons and the small instanton singularity is removed. In fact, this is the situation at generic point in the moduli space of the boundary CFT if Q_1 and Q_5 are coprime. In this situation we find that all branes in AdS_3 will have $T > q$. If Q_1 and Q_5 are large then we can find branes whose charges are closely aligned with Q_1, Q_5. In this case the action (3.7) becomes

$$S = T(\text{Area}) - 2q(\text{Volume}) = T(\cosh\rho)^2 - \frac{q}{2}\sinh 2\rho - q\rho \sim \frac{q}{4}[\epsilon e^{2\rho} - 4\rho + 2] \qquad (3.11)$$

with a small $\epsilon = \frac{T-q}{q} > 0$. We see that for large ρ the action (3.11) is positive. This means that the asymptotic boundary conditions are stable. We can mod out the theory by a $Z_N \subset \mathrm{U}(1)_L \subset \mathrm{SU}(2)_L \subset \mathrm{SO}(4)$, where SO(4) is the group of rotations of S^3. Then it is possible to remove all perturbative instabilities. The dual CFT corresponds to the theory we obtain if we consider D1 and D5 branes at an A_N singularity, which was studied in [23]. So in this case the two boundary solution is a perturbatively stable solution.

On the other hand, it is clear that for large Q_1, Q_5 it will be possible to find branes with small ϵ. If $\epsilon \ll 1$ then we see that the action (3.11) becomes negative for some intermediate values of ρ, even though it is positive for large ρ. So we get a non-perturbative instability under brane creation, i.e. we can decrease the action of the euclidean solution by creating brane/anti-brane pairs and moving them to a suitable position in the ρ coordinate where (3.11) is negative. Notice that we can only reduce the action by a finite amount through this process. So we expect to find another solution which will indeed be stable.

[10] See also [34] for more about 3-manifolds and their different quotients.

[11] Some lorentzian versions of these different quotients were described in [35].

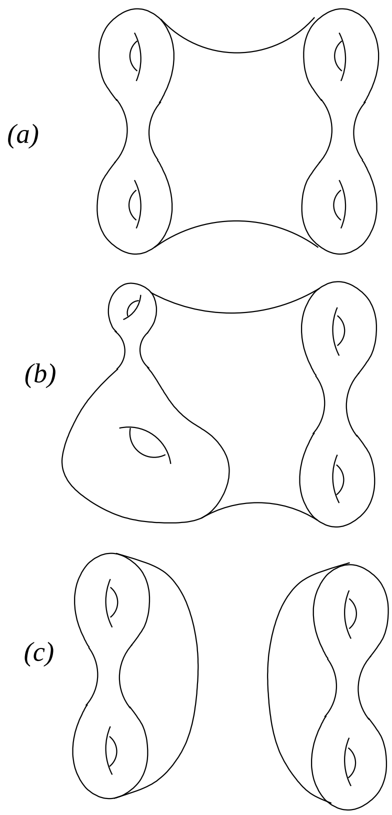

Figure 2: In (a) we see a three dimensional geometry with two boundaries. Both boundaries are identical Riemann surfaces. These results from modding out H_3 by a fuchsian group. In (b) we see a three dimensional manifold ending on two Riemann surfaces with different values of the moduli. These result from quotienting H_3 by a quasi-fuchsian group. Finally in (c) we see two disconnected three dimensional manifolds each ending on one boundary. Each of these manifolds results from quotienting H_3 by a Schottky group.

Of course this system also has the solutions depicted in figure 2c. But we did not compute the action difference between the solutions in figures 2a and 2c.

In summary, in this example we have a well defined conformal field theory which can live on any Riemann surface. We find a simple geometry that connects two Riemann surfaces. This geometry seems to be perturbatively stable, though it has non-perturbative instabilities under brane creation.

4. An example based on merons

In this section we construct wormholes that connect two AdS_4 regions that involve SU(2) gauge fields. These were considered in [12, 36, 37].[12] Here we will embed these solutions in M-theory and then we will discuss some aspects of the solution.

[12]In [37] it was argued that these meron configurations do not contribute in four flat dimensions. As we explain later, they do not contribute in AdS if we have the standard boundary conditions for the gauge fields. On the other hand they do contribute if we impose special, non-standard boundary conditions. We will consider this latter case below.

We start with eleven dimensional supergravity compactified on S^7. This has a well known solution of the form $AdS_4 \times S^7$ whose dual field theory is the theory on coincident $M2$ branes or IR limit of 2+1 Yang Mills with sixteen supercharges. The supergravity theory has a consistent truncation to an SO(8) gauged supergravity theory [38]. This in turn has a truncation to a theory with just an SU(2) gauge field and a graviton. This truncation can be understood as follows. We consider the spinor representation $\mathbf{8}_c$ of SO(8). SO(8) triality maps this to the vector $\mathbf{8}_v$ of SO(8). We consider the $\mathrm{SO}(3) \times \mathrm{SO}(5)$ subgroup that rotates the first three and the second five directions of this vector. The truncation consists in keeping all fields invariant under this SO(5). This projects out all the scalars of SO(8) gauged supergravity [39]. The bosonic fields are just the graviton and the Yang Mills fields. Another way to obtain this truncation is first to consider the SO(4) truncation studied in [40] and then further truncate it to the diagonal SU(2) in $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R = \mathrm{SO}(4)$. It is useful to understand how this SU(2) subgroup acts on the original vector representation of SO(8), i.e. the representation which the scalars on the M2 brane belong to. If we divide the 8 scalars into 4+4 we break SO(8) to $\mathrm{SO}(4) \times \mathrm{SO}(4)$. Each of the SO(4) factors is a product of two SU(2) groups. The SU(2) we are interested in is the diagonal combination of an SU(2) coming from each of the two SO(4). In other words the $\mathbf{8}_v$ of SO(8) transforms a pair of $\mathbf{2}$s of SU(2).

In summary, we end up with the action

$$S \sim N^{3/2} \int d^4x\sqrt{g}\ \left[-(R+6) + \alpha F^a_{\mu\nu} F^{a\mu\nu}\right] \tag{4.1}$$

with $\alpha = 1$. We have chosen units such that the radius of AdS is one for the solution with $F = 0$, and the gauge field is normalized so that the connection is given by $A = i\frac{\sigma^a}{2}A^a$ and $F = dA + A^2$.

4.1 A simple one boundary solution

If we take a gauge field such that $F = \pm * F$, then its stress tensor is zero and the geometry is still AdS_4. One way of viewing this solution is the following. AdS_4 is conformally flat. Since the self duality equations are independent of the conformal factor we can take a solution in flat space and translate it to a solution in AdS_4. There is, however, an important point. AdS_4 is conformal to the interior of a unit ball in R^4. This implies that with the generic instanton configuration the gauge field at the boundary of the unit ball will not vanish, more precisely, it will not be pure gauge. This means that the gauge field does not vanish at the boundary of AdS. Therefore the dual field theory is not the usual field theory on coincident M2 branes. It is the M2 brane field theory which has been coupled to a fixed background SU(2) gauge connection. This connection couples to the SU(2) currents which are part of the SO(8) global symmetry currents of the theory. In appendix A.6 we write the action for a single M2 brane.[13] Even though an instanton preserves supersymmetry in flat space, once we are in AdS it no longer preserves supersymmetry. Furthermore,

[13]There is no lagrangian for multiple M2 branes. It is not clear how to write a lagrangian for 2+1 super YM that will flow to the one we are considering since the SU(2) subgroup we are dealing with is not contained in SO(7), which is the symmetry group of $2+1$ SYM with 16 supercharges.

JHEP02(2004)053

the boundary conditions themselves break supersymmetry. So we are dealing with a non-supersymmetric deformation of the coincident M2 branes theory. In appendix A.5 we prove these assertions.

If we consider a single instanton at the center of R^4, this gives us a configuration that is spherically symmetric, up to gauge transformations. The gauge field can be written as

$$A^a = f w^a \tag{4.2}$$

where w^a are the left invariant one forms on S^3 and f is a function of the radial coordinate, see appendix A.1 for more details. The self duality conditions for the instanton imply that in R^4 f obeys a first order equation that is solved by setting $f^{-1} = 1 + \frac{r_0^2}{r^2}$, where r is the radial coordinate on R^4. In terms of the AdS_4 metric $ds^2 = d\rho^2 + \sinh^2 \rho ds^2_{S^3}$ we find that

$$f(\rho) = \frac{f_B \sinh^2 \rho/2}{\cosh^2 \rho/2 - f_B} \,. \tag{4.3}$$

When $\rho \to \infty$ this asymptotes to the boundary value f_B. When $f_B = 1/2$ we have precisely half an instanton in AdS.

For each value of the boundary condition, f_B, there are at least two possible gauge field configurations in the interior. One corresponding to a instanton and one corresponding to an anti-instanton in R^4 with the same boundary value of the fields at the unit sphere in R^4. The action for these self dual configurations can be computed in terms of the instanton charge inside AdS and it is equal to

$$S_{sd}[f_B] = \alpha \mathcal{N} 16 f_B^2 (3 - 2 f_B) \tag{4.4}$$

for the self dual configuration, where $\mathcal{N}$ is an overall factor common to all computations we are doing.[14] For the anti-self dual configuration we find $S_{asd}[f_B] = S_{sd}[1 - f_B]$. In the particular case $f_B = 1/2$ both are equal. The gravity part of the action is equal to $S_{\rm grav} = 4\mathcal{N}$ after subtracting the usual counter terms, see appendix A.2. So the total action for the $f_B = 1/2$ case is

$$S_{1\,\rm bdy} = (4 + 8\alpha)\mathcal{N} \,. \tag{4.5}$$

An interesting configuration is the zero size instanton. In this case it seems that the solution might be supersymmetric (we are not sure because we cannot trust supergravity). It is also a solution with the same boundary conditions as the usual $AdS_4 \times S^7$ solution. It would be very nice to figure out the role that these zero size instantons play in the physics of $AdS_4 \times S^7$. If we start with $AdS_4 \times S^7$ with $f_B = 0$ and we increase continuously f_B to $f_B = 1$, then we end up with a zero size instanton in the interior. On the other hand $f_B = 1$ is gauge equivalent to $f_B = 0$. So this sum over zero size instantons is necessary to make sure that the field theory is invariant under large gauge transformations.

[14] $\mathcal{N}$ is equal to the normalization factor of (4.1) outside the integral times the volume of S^3, $\Omega_3 = 2\pi^2$.

4.2 A two-boundary solution

Now let us consider the situation where the boundary values of the gauge fields are given by (4.2) with $f_B = 1/2$. It turns out that in this case we can look for a solution with $f(\rho) = 1/2$. This will solve the Maxwell equations, as long as the metric is SO(4) symmetric. This is not a self dual configuration, so the stress tensor is non-zero and one should solve Einstein's equations. We find that the solution is given by

$$ds^2 = d\rho^2 + e^{2\omega} ds^2_{S^3}\,, \qquad e^{2\omega} = \sqrt{\alpha + \frac{1}{4}}\cosh 2\rho - \frac{1}{2}\,. \tag{4.6}$$

In this solution the three sphere never shrinks completely, it has a minimum size at $\rho = 0$ and it increases as $\rho \to \pm\infty$.

A natural question is whether this two boundary solution is stable under small perturbations. A good candidate for an unstable mode is a mode of the form

$$A = \left[\frac{1}{2} + \epsilon(\rho)\right] g dg^{-1} \qquad \text{with} \lim_{t\to\pm\infty} \epsilon(\rho) = 0 \tag{4.7}$$

where ϵ is small, and g is an SU(2) group element, such that $i\frac{\sigma^a}{2}w^a = g^{-1}dg$. Inserting this into the euclidean action and expanding to second order in ϵ we find that indeed there is a negative mode. See appendix A.3 for details.

It is interesting to find the action for this configuration. For $\alpha = 1$ it approximately given by $S = (30.296\ldots)\mathcal{N}$. This action is larger than twice the action of the one boundary solution (4.5). This suggests that the instability we saw above will deform the two boundary solution into two disconnected two boundary solutions.

It is interesting to note that for other values of α in the action (4.1) the physics is different. If $\alpha > 1.245 \pm .005$ then the two boundary solution is perturbatively stable. Furthermore the action of the two boundary solution will be smaller than the action of two one boundary solutions when $\alpha > 3.775 \pm .005$. We do not know of a configuration in string theory that would give (4.1) with these large values of α.

5. An example involving $N = 4$ Yang Mills

In this section we will discuss an example that is physically rather similar to the one we found above. This example will produce again a two boundary solution, however in this case we do not find any negative modes around the solution, so it might be a local minimum of the euclidean functional integral. The basic idea is to start with the standard $AdS_5 \times S^5$ example of AdS/CFT. Suppose we could truncate the supergravity theory to a 5 dimensional theory involving an SU(2) gauge field and the metric. We could consider a configuration containing an instanton on the S^4 boundary of AdS. This would lead to a two boundary solution similar to the one we had above. It turns out that it is not possible to perform this truncation due to the presence of Chern Simons terms in the five dimensional theory. Instead, we will consider the consistent KK reduction to five dimensions considered in [41]. This is a reduction that keeps the SO(6) gauge fields and some of the scalar fields parameterizing deformations of the five-sphere. The reduced action is

$$\mathcal{L}_5 = R*1 - \frac{1}{4}T^{-1}_{IJ} * DT_{JK} \wedge T^{-1}_{KL} DT_{LI} - \frac{1}{4}T^{-1}_{IK}T^{-1}_{JL} * F^{IJ} \wedge F^{KL} - V*1 - \tag{5.1}$$
$$- \frac{1}{48}\epsilon_{I_1\cdots I_6}\left(F^{I_1I_2}F^{I_3I_4}A^{I_5I_6} - gF^{I_1I_2}A^{I_3I_4}A^{I_5J}A^{JI_6} + \frac{2}{5}g^2A^{I_1I_2}A^{I_3J}A^{JI_4}A^{I_5K}A^{KI_6}\right)$$

where A^{IJ} are the SO(6) gauge fields, T^{IJ} is a 6×6 symmetric unimodular matrix of scalars and the potential V is given by $V = \frac{1}{2}g^2(2T_{IJ}T_{IJ} - (T_{II})^2)$.

Consider now the $\mathrm{SO}(3)\times\mathrm{SO}(3)$ subgroup of SO(6) that rotates the first three coordinates and the second three coordinates of R^6. We consider a configuration that consists of an instanton on S^4 for the first SO(3) and an anti-instanton for the second SO(3). We further consider instantons and anti-instantons that are SO(5) symmetric under rotations of S^4. This gauge field configuration sets the boundary conditions at the boundary of AdS_5. As in the example we considered above, the corresponding gauge theory is $\mathcal{N}=4$ SYM with an external fixed gauge field coupled to the SO(6) currents. In this case the field theory itself, as defined on each boundary seems stable and well defined.

Let us describe more concretely the two boundary solution. We consider a five dimensional space which is SO(5) symmetric and it is foliated by S^4s. In other words, the metric has the form

$$ds^2 = d\rho^2 + e^{2\omega}ds^2_{S^4} = d\rho^2 + e^{2\omega}\left(d\theta^2 + \sin^2\theta\,\frac{1}{4}w^aw^a\right) \tag{5.2}$$

where w^a are the left invariant one forms on S^3. We pick a gauge field configuration given by

$$A^{IJ}_\mu = iA^a_\mu L^{a\,IJ} + i\tilde{A}^a_\mu \tilde{L}^{a\,IJ} \tag{5.3}$$

where L^a and $\tilde{L}^a$ are the generators of $\mathrm{SO}(3)\times\mathrm{SO}(3)$. We pick a gauge field A^a to be an instanton and $\tilde{A}^a$ to be an anti-instanton which are SO(5) symmetric under rotations of S^4

$$A^a = \cos^2\frac{\theta}{2}w^a\,, \qquad \tilde{A}^a = \sin^2\frac{\theta}{2}w^a\,. \tag{5.4}$$

Let us check that this is a configuration which solves the equations of motion for the scalars in the **20** of SO(6) that couple to the Yang Mills fields. The linearized coupling to these scalars has the form

$$\phi_{IJ}F^{IK}_{\mu\nu}F^{KJ\,\mu\nu} \tag{5.5}$$

where the indices of ϕ_{IJ} are symmetric and traceless. On the other hand, the configuration under consideration was designed so that $F^{IK}_{\mu\nu}F^{KJ\,\mu\nu} \sim \delta^{IJ}$. Then (5.5) vanishes.

The equation of motion for the Yang Mills fields involve a term coming from the F^2 term in the lagrangian and one term coming from the Chern-Simons coupling in (5.1): $\epsilon_{I_1\cdots I_6}F^{I_1I_2}\wedge F^{I_3I_4}\wedge A^{I_5I_6}+\cdots$. This leads to a term in the equations of motion for A^{IJ} of the form

$$\frac{\delta S_{CS}}{\delta A^{IJ}} \sim \epsilon_{IJKLMN}F^{KL}\wedge F^{MN}\,. \tag{5.6}$$

Our choice of instanton and anti-instanton configuration has

$$F^{KL} \wedge F^{MN} \sim \left(L^{a\ KL} L^{a\ MN} - \tilde{L}^{a\ KL} \tilde{L}^{a\ MN} \right) \epsilon_4 \,. \tag{5.7}$$

When this is inserted in (5.6) we get zero. See appendix A.7 for details. Note that (5.7) implies that the total SO(6) instanton number is zero.

If we were to choose $\tilde{F}^a = F^a$ then we would get a cross term in (5.7) that involves $L^{a\,KL} \tilde{L}^{a\,MN}$ which would lead to a non-zero right hand side in (5.6). This choice of gauge fields leaves an unbroken U(1) that rotates the first three coordinates in R^6 into the second three coordinates of R^6. The nonzero result for (5.6) is implying that the instantons generates a charge under this U(1). It might be possible to add other charged particles to the system which will cancel this extra term. This seems to complicate the analysis and we have not explored this possibility.

Now we need to find the five dimensional geometry. We can write the effective action for ω in (5.2)

$$S \sim \int d\rho e^{4\omega} \left(-\omega'^2 - e^{-2\omega} - 1 + \beta e^{-4\omega} \right) \tag{5.8}$$

where in our case $\beta = 1$. We need to consider the zero energy solution, which turn out to be identical to the one in the previous section. So the metric is given again by

$$e^{2\omega} = \sqrt{\beta + \frac{1}{4}} \cosh 2\rho - \frac{1}{2} \,. \tag{5.9}$$

Note that the gauge field on S^4 is topologically trivial as an SO(6) gauge field. In fact, in appendix A.8 we describe a continuous deformation which transforms the instanton/anti-instanton configuration into a configuration which is a gauge transformation of the zero gauge field configuration. So we expect that if we consider fluctuations of the gauge fields that carry indices under both SO(3) groups we will get negative modes. In fact, if we consider the SO(6) gauge theory on S^4, we can see that the instanton/anti-instanton configuration has some negative modes. We find a negative mode explicitly in appendix A.8. However, it turns out that when we include the radial direction and we demand that fluctuations are normalized at $\rho = \pm\infty$, then we find that the negative mode on S^4 does not give rise to negative modes in the full five dimensional geometry.

We also studied possible negative modes coming from the scalars in the **20** of SO(6) and we found that there were none, see appendix A.9. The conformal factor of the metric does not lead to negative modes either, once we Wick rotate it as in [16]. In principle we should check all other fields of the ten dimensional supergravity theory to check that there are no negative modes coming from other fields. We only analyzed the fields that we thought were natural candidates for negative modes.

In this case it is also likely that there is a single boundary solution, but we did not find it explicitly.

6. Discussion

In this paper we have considered a variety of euclidean solutions which connect two boundaries. These pose a puzzle from the point of view of AdS/CFT. From the field theory point

JHEP02(2004)053

of view the correlation functions across the two boundaries should factorize, while from the gravity point of view they do not. So we can see two possibilities. Either one can introduce a subtle correlation between the two field theories or the full quantum gravity answer, after summing over all geometries, is such that the gravity correlators indeed factorize.

We have emphasized the presence or absence of perturbative negative modes. The simplest case is when there are no perturbative negative modes. Then the two boundary geometry represents a local minimum of the action. If a negative mode exists, then there are nearby configurations which might contribute in a more important way to the partition function than the configuration we started with. In this case it is less clear that there is indeed a correlation between the two boundaries (thought it is also not clear that there is no correlation).

If we take two decoupled field theories then we are instructed to sum over all manifolds that end on the respective boundaries. Why should we forbid the manifolds that connect the two boundaries? If the two field theories are truly decoupled we would conclude that the final effect of performing the sum over all manifolds that connect the two boundaries would completely factorize into the two partition functions of the two field theories.

In the context of ordinary wormholes the corresponding problem is the fact that the wormholes seem to induce non-local interactions that would lead to violations of unitarity in the lorentzian theory [42, 43, 44]. By an "ordinary wormhole" we mean a wormhole such as the axionic wormhole of [7] which could connect any two points in spacetime. In this context a possible solution was suggested by Coleman [45]. He suggested that the effect of the sum over all possible wormholes leads to superselection sectors characterized by some parameters α_i. These parameters characterize the wavefunction of the closed universes associated to wormholes. An observer would see local and unitary physics but she would not be able to compute the parameters α_i from first principles, she would have to measure them experimentally. An axionic wormhole then would look like an ordinary instanton effect that breaks the U(1) translation symmetry of the axion. In an AdS solution to string theory where axionic wormholes existed one would be forced to conclude that the local field theory at the boundary of AdS corresponds to the theory with fixed α_i parameters, i.e. to a given superselection sector.[15] Otherwise the integral over α_i would induce non-local effects in the field theory.

It is tempting to speculate that one could have a similar situation with the wormholes that we found in our paper. These wormholes would have a large number of associated α parameters. The number of parameters is the dimension of the Hilbert space for the associated compact universe. This can be estimated as a typical area in the compact universe, which in our cases goes as the central charge of the corresponding field theory.[16] This would give a large number of parameters for large N. In Coleman's arguments it was important that the wormholes could exist with arbitrary wavefunctions and this leads to

[15]We do not know of a concrete nonsingular string theory AdS example where axionic wormholes actually do exist.

[16]In the supergravity approximation all definitions of the central charge give similar values. In the examples of sections 3, 4, 5 this goes as $c \sim Q_1 Q_5$, $N^{3/2}$, N^2 respectively. The number of states then would be of the order of e^c.

an indeterminacy of the α parameters [46]. If one thinks that the field theories that we introduced correspond to particular values of α parameters, then it is hard to see which deformations of the field theories would lead to other values.[17] So it seems most probable that in quantum gravity the wavefunction for these closed universes is completely determined. To the extent that the field theory on the boundary determines the α parameters, then the wavefunction for the closed universes are also determined by the field theory. It would be nice to understand to what extent the field theory contains some information about these closed universes.[18]

On the other hand, it might be that geometries with two boundaries correspond to a partition function of the form $Z = \sum_i Z_i^1 Z_i^2$ where the index i runs over some "sector" of the field theory and the indices $1, 2$ indicate the two field theories at each boundary. Such a situation might arise if the field theory partition functions are not well defined.[19] An example of such a theory is a chiral boson in two dimensions. One way to define a modular invariant answer is to consider a left and right moving sector. It was observed in [49] that the $AdS_5 \times S^5$ partition function has a similar property. On the other hand the effects discovered in [49] are related to the overall U(1) degree of freedom. So by adding a simple U(1) field at the boundary one can get a well defined partition function.[20]

In summary, a better understanding of the physics associated to these geometries is needed. This might lead to interesting insights on how quantum gravity works.

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A. Details about solutions involving gauge fields

A.1 Generalities about merons and instantons

Let us consider an SU(2) group element $g = e^{\frac{i}{2}\phi\sigma^3} e^{\frac{i}{2}\theta\sigma^1} e^{\frac{i}{2}\psi\sigma^3}$, this parameterizes an S^3

$$ds^2_{S^3} = -\frac{1}{2}\,\mathrm{Tr}(g^{-1}dgg^{-1}dg) = \frac{1}{4}\left[(d\psi + \cos\theta d\phi)^2 + d\theta^2 + \sin^2\theta d\phi^2\right] = \frac{1}{4} w^a w^a\,. \qquad (A.1)$$

Where we defined the left invariant one forms w^a through $i\frac{\sigma^a}{2} w^a = g^{-1}dg$. We define the SU(2) gauge field $A = i\frac{\sigma^a}{2} A^a$. The field strength is $F = dA + A^2$ or $F^a = dA^a - \frac{1}{2}\epsilon^{abc} A^b \wedge A^c$. We will be interested in solutions where $A = f(r) g^{-1}dg$, or $A^a = f w^a$. If $f = 1$ then we have

[17] See [47] for a similar discussion.

[18] Hopefully, the techniques of [48] could be extended to probe the singularities of the lorentzian closed universes.

[19] The boundary field theories discussed in sections 4, 5 seem perfectly well defined in this sense. In the (0,4) 2d CFT associated with the $AdS^3 \times S^3/Z_N \times K3$ example in section 3 there could be subtleties due to the fact that the theory is not left/right symmetric.

[20] If one imposes local boundary conditions for the NS and RR two form potentials, then one recovers this U(1) degree of freedom [50].

a pure gauge configuration. The field strength is $F = f'dr \wedge g^{-1}dg + f(f-1)g^{-1}dg \wedge g^{-1}dg$. In four flat dimensions we find that the self duality condition implies $f^{-1} = 1 + \frac{r_0^2}{r^2}$. In order to find the self dual configuration in AdS_4 we start with the metric as $ds^2 = d\rho^2 + \sinh^2 \rho ds^2_{S^3}$. We see that defining $dr/r = d\rho/\sinh\rho$, i.e.

$$r = \tanh\frac{\rho}{2} \tag{A.2}$$

we can make the AdS metric conformally flat. This implies that the self dual configuration has $f^{-1} = 1 + C/\tanh^2\rho/2$. Note that as ρ runs from zero to infinity r goes from zero to one. This implies that AdS_4 is conformal to the unit ball in R^4. If we are interested in instantons on S^4 all we need to do is to change $\rho \to i\gamma$, $0 \leq \gamma \leq \pi/2$ in the above expressions. Then the function f becomes $f^{-1} = 1 + C'/\tan^2\gamma/2$ and for $C' = 1$ we find that $f = \sin^2\gamma/2$. In order to get anti-self dual configurations all we just change $f \to 1 - f$.

For a general metric

$$ds^2 = d\rho^2 + e^{2\omega(\rho)}ds^2_{S^3} \tag{A.3}$$

the effective action for f becomes

$$\begin{aligned} S &= \alpha \int F^a_{\mu\nu}F^{a\,\mu\nu} = 24\alpha\Omega_3 \int d\rho e^{-\omega}\left[e^{2\omega}(\partial_\rho f)^2 + 4f^2(1-f^2)\right] = \\ &= 24\alpha\Omega_3 \int dy[(\partial_y f)^2 + 4f^2(1-f^2)] \end{aligned} \tag{A.4}$$

where $dy = d\rho e^{-\omega(\rho)}$, and where Ω_3 - the volume of a three sphere.

We are interested in evaluating this action for the following three cases. First when (A.3) is AdS_4 and the solution for f is that of a self dual configuration such that the value of f at the boundary of AdS_4 is f_B. Then we find that (A.4) is

$$S_{sd} = 16\alpha\Omega_3 f_B^2(3 - 2f_B) = 8\alpha\Omega_3\left[1 + 3\left(f_B - \frac{1}{2}\right) - 4\left(f_B - \frac{1}{2}\right)^3\right]. \tag{A.5}$$

For an anti-self dual configuration with the same boundary conditions we find

$$S_{\text{anti-sd}} = 16\alpha\Omega_3[-f_B^2(3 - 2f_B) + 1]\,. \tag{A.6}$$

Of course we can get (A.6) from (A.5) by replacing $f_B \to 1 - f_B$. For $f_B = 1/2$ both have the same action, but for $f_B < 1/2$ the self-dual configuration has lower action than the anti-self-dual configuration. Note that the instanton number inside AdS is less than one. For $f_B = 1/2$ it is $1/2$.

In the case that we have an instanton on S^4 the action is the usual $S_{\text{inst}} = \alpha \int F^a_{\mu\nu}F^{a\,\mu\nu} = 16\alpha\Omega_3 = 32\pi^2\alpha$.

A.2 Four dimensional solutions

The renormalized gravity action is

$$S_{GR} = \int d^4x\sqrt{g}\,[-(R+6)] - 2\int d^3x\sqrt{\gamma}\,K + \left(4\int d^3x\sqrt{\gamma} + \int d^3x\sqrt{\gamma}\,R_\gamma\right) \tag{A.7}$$

where γ is the metric at the regularized boundary, R_γ is its curvature and K is the extrinsic curvature of the regularized surface [51]. If the metric is AdS_4, i.e. of the form (A.3) with $e^\omega = \sinh(\rho)$ we find that (A.7) is equal to $S_{GR} = 4\Omega_3$, when we take away the regulator. So the total action for the one boundary solution is

$$S_{1\,\mathrm{bdy}} = (4 + 8\alpha)\Omega_3 \,. \tag{A.8}$$

After multiplying by the overall normalization factor of the action we get (4.5).

Now we consider the case where $f = 1/2$ which leads to the two boundary solution. For a metric of the form (A.7) one can write the gravity action (A.7) plus the Yang-Mills contribution as

$$S \sim 6 \int^{\rho_M} d\rho \left(-\omega'^2 - e^{-2\omega} - 1 + \alpha e^{-4\omega}\right) e^{3\omega} + 4e^{3\omega_M} + 6e^{\omega_M} \tag{A.9}$$

where α is the coefficient of the gauge kinetic term and $w_M \equiv w(\rho_M)$. As usual, we should look for a zero energy solution of this action, which implies

$$\omega'^2 - e^{-2\omega} - 1 + \alpha e^{-4\omega} = 0 \,. \tag{A.10}$$

Solving this equation we find (4.6). We can now define a new variable $x = e^\omega$, use (A.10) and write the counter terms in (A.9) as total derivatives to find that the two boundary action is

$$\begin{aligned} S_{2\,\mathrm{bdy}} &= 12 \left[\int_{x_0}^{\infty} dx \left(-2\sqrt{x^4 + x^2 - \alpha} + 2x^2 + 1\right) + \frac{2}{3}x_0^3 + x_0 \right] \Omega_3 \\ &= (30.296\ldots)\Omega_3 \qquad \text{for } \alpha = 1 \end{aligned} \tag{A.11}$$

where x_0 is the value of x at $\rho = 0$,

$$x_0^2 = \sqrt{\alpha + \frac{1}{4}} - \frac{1}{2} \,. \tag{A.12}$$

It is now interesting to consider the action difference $S_{2\,\mathrm{bdy}} - 2S_{1\,\mathrm{bdy}}$. We find that for the example coming from M-theory, when $\alpha = 1$, this difference is positive. But for sufficiently large α ($\alpha > 3.775 \pm .005$) this becomes negative. In such theories the two boundary solution has less action. It is not clear if one can find an example coming from string theory that has α large enough.[21]

A.3 Stability of the 2-boundary solution

We now consider small fluctuations of the two boundary solution of the form $f = 1/2 + \epsilon(\rho)$ with $\epsilon \ll 1$. We substitute this in (A.4) and we find

$$S_{\mathrm{quad}} = 24 \int_{-y_m}^{y_m} dy \left[(\partial_y \epsilon)^2 - 2\epsilon^2\right] \tag{A.13}$$

[21] One case we could consider is the squashed S^7 of [52] this leads to a theory in four dimensions that contains SO(3) gauge bosons, but it is not clear whether one can truncate it to pure 4d gravity plus the SO(3) gauge field.

JHEP02(2004)053

where

$$y_m = \int_0^\infty d\rho e^{-w} = \int_0^\infty d\rho w' e^{\omega} \frac{e^{-2\omega}}{w'} = \int_{x_0}^\infty \frac{dx}{\sqrt{x^4 + x^2 - \alpha}} \tag{A.14}$$

and $x_0 \equiv \sqrt{\alpha + \frac{1}{4}} - \frac{1}{2}$. The eigenvalue problem for this solution is very simple, we impose that ϵ vanishes at $y = \pm y_m$. The lowest mode has the form $\cos(\pi y/(2y_m))$ and it leads to the eigenvalue

$$\lambda_0 = -2 + \left(\frac{\pi}{2y_m}\right)^2 . \tag{A.15}$$

For $\alpha = 1$ this is negative. For $\alpha > 1.245 \pm 0.005$ this is positive.

A.4 The M-theory lift of the solutions

Using the formulas in [40] with the two equal gauge fields, $A^a = \tilde{A}^a$, we get the 11-dimensional solution

$$\begin{aligned} ds^2_{11} &= ds^2_4 + 4d\xi^2 + \cos^2\xi \sum_a (w^a - A^a_\mu dx^\mu)^2 + \sin^2\xi \sum_a (\tilde{w}^a - A^a_\mu dx^\mu)^2 \\ F_{(4)} &= -3\epsilon_{(4)} + \sqrt{2}\sin\xi\cos\xi d\xi \wedge (w - \tilde{w})^a \wedge *F^a_2 + \\ &\quad + \frac{\sqrt{2}}{4}\epsilon_{abc}\left[\cos^2\xi (w - A)^a \wedge (w - A)^b + \sin^2\xi (\tilde{w} - A)^a \wedge (\tilde{w} - A)^b\right] \wedge *F^c_2 \end{aligned} \tag{A.16}$$

where w^a, $\tilde{w}^a$ are SU(2) right invariant one forms on two 3-spheres S^3, $\tilde{S}^3$. This solution does not display the full SO(5) symmetry that is present when the two SU(2) gauge fields of [40] are equal. We can write it in an SO(5) symmetric fashion by thinking about S^7 as an S^3 fibration over S^4. This can be done by writing the metric in (A.16) in the following way. We parameterize S^3 in terms of the SU(2) group element g and $\tilde{S}^3$ in terms of the SU(2) group element $\tilde{g}$. Then we write the right invariant one forms as $\frac{i\sigma^a}{2} w^a_R = g dg^{-1}$ and $\frac{i\sigma^a}{2}\tilde{w}^a_R = \tilde{g} d\tilde{g}^{-1}$ where σ^a are the Pauli matrices. We then define $\hat{g} = \tilde{g}^{-1} g$ with $\frac{i\sigma^a}{2}\hat{w}^a_R \equiv \hat{g} d\hat{g}^{-1}$, so that $g = \tilde{g}\hat{g}$ and $\tilde{g}^{-1} g dg^{-1}\tilde{g} = d\tilde{g}^{-1}\tilde{g} + \hat{g} d\hat{g}^{-1}$ and $w^2_R = -2\,\mathrm{Tr}(g dg^{-1})^2 = (\tilde{w}_L + \hat{w}_R)^2$. The S^7 metric (of radius 2) is then

$$\begin{aligned} ds^2 &= 4d\xi^2 + \cos^2\xi w^2_R + \sin^2\xi \tilde{w}^2_R \\ &= \left[d\zeta^2 + \sin^2\zeta \frac{1}{4}\hat{w}^2_R\right] + \left(\tilde{w}_L + \cos^2\frac{\zeta}{2}\hat{w}_R\right)^2 \end{aligned} \tag{A.17}$$

where we used that $\tilde{w}^2_L = \tilde{w}^2_R$ and we defined $\zeta \equiv 2\xi$, $0 \le \zeta < \pi$. The coordinates ζ and $\hat{g}$ parameterize an S^4. We see that $B^a \equiv \cos^2\frac{\zeta}{2}w^a_R$ can be thought of as the gauge field of an SU(2) instanton on S^4 that is SO(5) symmetric, up to "gauge" transformations. The gauge transformations that act on the gauge field B are the right SU(2) transformations acting on the fiber parameterized by $\tilde{g}$.

Now we are ready to add the A gauge field appearing in the metric in (A.16). We see that $-2\,\mathrm{Tr}(w_R - A)^2 = (\tilde{w}_L + \hat{w}_R - \tilde{g}^{-1}A\tilde{g})^2$, where hopefully the notation is self explanatory.[22] The metric then becomes

$$\begin{aligned} ds^2 &= \left[d\zeta^2 + \frac{1}{4}\sin^2\zeta\hat{w}_R^2\right] + (\tilde{w}_L + B)^2 - \\ &\quad -2\cos^2\xi\,\mathrm{Tr}(\tilde{w}_L\tilde{g}^{-1}A\tilde{g}) - 2\cos^2\xi\,\mathrm{Tr}(\hat{w}_R\tilde{g}^{-1}A\tilde{g}) + A^2 - 2\sin^2\xi\,\mathrm{Tr}(\tilde{w}_R A) \\ &= \left[d\zeta^2 + \frac{1}{4}\sin^2\zeta\hat{w}_R^2\right] + (\tilde{w}_L + B)^2 + A^2 - 2\,\mathrm{Tr}(\tilde{w}_L\tilde{g}^{-1}A\tilde{g}) - 2\cos^2\xi\,\mathrm{Tr}(\tilde{g}\omega_R\tilde{g}^{-1}A) \\ &= \left[d\zeta^2 + \frac{1}{4}\sin^2\zeta\hat{w}_R^2\right] + (\tilde{w}_L + B - \tilde{g}^{-1}A\tilde{g})^2 \\ &= \left[d\zeta^2 + \frac{1}{4}\sin^2\zeta\hat{w}_R^2\right] + (\tilde{w}_R - A + \tilde{g}B\tilde{g}^{-1})^2\,. \end{aligned} \tag{A.18}$$

We see that the two gauge fields B and A are associated to the right and left rotations of the $\tilde{S}^3$ fiber parameterized by $\tilde{g}$. The final geometry corresponds to fibering the $\tilde{S}^3$ over $S^4 \times M^4$ with a gauge field which is the sum of the instanton, B, on S^4 plus the gauge field, A, on the four dimensional manifold M^4.

A.5 Checking that this solution is not supersymmetric

A solution that contains a self dual gauge field configuration on H_4 looks like a supersymmetric configuration, since instantons are usually associated to supersymmetric configurations. In this case it is possible to check that the solution is not supersymmetric.

In order to do this analysis it is useful to consider the supersymmetries of SO(4) gauged supergravity given in [53, 54]. In this theory, apart of the bosonic fields discussed in [40] there are also the following fermionic fields: four ψ_μ^i spin 3/2 Majorana spinors with a vector index in SO(4), and 4 spin 1/2 Majorana spinors χ^i.[23]

Also we should separate F^{ij} to the two SU(2) fields — we do this in the usual way: $J^{12} + J^{34} \equiv iL^1$, $J^{12} - J^{34} \equiv i\tilde{L}^1$, $J^{13} - J^{24} \equiv iL^2$ etc. (so that $[L^a, L^b] = i\epsilon^{abc}L^c$,same for the tildes and the L^a, $\tilde{L}^a$ commute among themselves). This is such that $A^{ij}J^{ij} = iA^aL^a + i\tilde{A}^a\tilde{L}^a$, where $A^a, \tilde{A}^a$ are real.

In the background we are interested in we have $\phi = \chi = 0$. Then the (lorentzian) susy transformations for the spinors become

$$\begin{aligned} \delta\bar{\chi}^i &= \frac{1}{2\sqrt{2}}\epsilon^{ijkl}\bar{\epsilon}^j\gamma^{\mu\nu}F_{\mu\nu}^{kl} \\ \delta\bar{\psi}_\lambda^i &= \bar{\epsilon}^i\overleftarrow{D}_\lambda - \frac{i}{2}\bar{\epsilon}^j\gamma_\lambda\gamma^{\mu\nu}F_{\mu\nu}^{ij} + ig\bar{\epsilon}^i\gamma_\lambda \end{aligned} \tag{A.19}$$

where the covariant derivative is: $D_\lambda\chi^k = ((\partial_\lambda + \frac{1}{2}\omega_{\lambda,ab}\gamma^{ab})\delta^{kl} + 2gA_\lambda^{kl})\chi^l$. Let us first take the dagger of these and multiply by γ_0 from the left:

$$\begin{aligned} \delta\chi^i &= -\frac{1}{2\sqrt{2}}\epsilon^{ijkl}F_{\mu\nu}^{jk}\gamma^{\mu\nu}\epsilon^l \\ \delta\psi_\lambda^i &= D_\lambda\epsilon^i - \frac{i}{2}F_{\mu\nu}^{ij}\gamma^{\mu\nu}\gamma_\lambda\epsilon^j - ig\gamma_\lambda\epsilon^i\,. \end{aligned} \tag{A.20}$$

[22]The expression $\tilde{g}^{-1}A\tilde{g}$ means that we rotate the gauge indices, a, of A_μ^a by the group element $\tilde{g}$.

[23]In order to match conventions in [53, 54] to the ones we used, which are the ones in [40] we set $\kappa = 1$, set $g_+ = 2g, g_- = 0$, relate A, B to ϕ, χ by $W = -A + iB = e^{i\sigma}\tanh(\lambda/2)$ with $\cosh\lambda = \cosh\phi + \frac{1}{2}\chi^2 e^\phi$, $\sinh\lambda\cos\sigma = \sinh\phi - \frac{1}{2}\chi^2 e^\phi$, $\sinh\lambda\sin\sigma = \chi e^\phi$.

Changing from SO(4) to SU(2) × SU(2) language, and setting the two SU(2) gauge fields to be equal: $A = \tilde{A}$ so that $F^{23} = F^{24} = F^{34} = 0$, we can write the susy equations in the following way, decomposing the SO(4) $\epsilon^{1,2,3,4}$ into the SU(2): η, ϵ^i, $i = 1, 2, 3$:

$$\begin{aligned} 0 &= F^i_{\mu\nu}\gamma^{\mu\nu}\epsilon^i \\ 0 &= F^i_{\mu\nu}\gamma^{\mu\nu}\eta \\ 0 &= (\nabla_\lambda - ig\gamma_\lambda)\eta \\ 0 &= (\nabla_\lambda - ig\gamma_\lambda)\epsilon^i - \varepsilon^{ijk}(F^j_{\mu\nu}\gamma^{\mu\nu}\gamma_\lambda + 4igA^j_\lambda)\epsilon^k\,. \end{aligned} \tag{A.21}$$

Now we perform in the standard way a Wick rotation from the lorentzian to euclidean signature [55]. One finds that the equations retain the same form in euclidean space.

We see that the equations for ϵ and η in (A.21) decouple. Consider first the conditions on η: the gravitino variation implies that η is a usual AdS_4 Killing spinor. Then we need to further impose that $F^i_{\mu\nu}\gamma^{\mu\nu}\eta = 0$. If we have a self-dual field then $F_{\mu\nu}\gamma^{\mu\nu} = \frac{1}{2}(F_{\mu\nu} + *F_{\mu\nu})\gamma^{\mu\nu} = F_{\mu\nu}\gamma^{\mu\nu}P_L$. It turns out that this condition implies the chirality condition $\Gamma^5\eta = \eta$. One can check that this is not compatible with the conditions for an AdS_4 Killing spinor. Note however that in the limit that the instanton has zero size this chirality projection condition is imposed only at the location of the zero size instanton. This is compatible with the Killing spinor equations in AdS. So it appears that a zero size instanton in AdS_4 is supersymmetric.

Now we consider supersymmetries generated by ϵ. Before doing this, note that there is a truncation of SO(4) gauged supergravity to an SO(3) gauged theory with 3 supersymmetries (the ungauged version of this SO(3) theory was considered in [56]). This theory contains the metric, three gravitons, three gauge fields, and one fermion. The supersymmetry variation of the fermion and gravitino are given by the ϵ dependent terms in (A.21).

First note that as $F_{\mu\nu}\gamma^{\mu\nu} = F_{\mu\nu}\gamma^{\mu\nu}\frac{1}{2}(1 - \Gamma^5)$, the first equation in (A.21) implies that $\gamma^{\rho i}\epsilon^i = 0$. If we use notations where $\epsilon^i_\pm \equiv \frac{1}{2}(1 \mp \Gamma^5)\epsilon^i$ this means:

$$\sigma^i\epsilon^i_\pm = 0 \tag{A.22}$$

where σ^i are the pauli matrices. Then the last equation in (A.21) becomes

$$\begin{aligned} \partial_\rho\epsilon^i_- - ig\epsilon^i_+ &= 0 \\ \partial_\rho\epsilon^i_+ - ig\epsilon^i_- &= -4ie^{-2w}f(1-f)\epsilon^{ijk}\sigma^j\epsilon^k_+ \\ D^{ij}_\alpha\epsilon^j_- &= -g\sigma_\alpha\epsilon^i_+ \\ D^{ij}_\alpha\epsilon^j_+ &= -4ie^{-2w}f(1-f)\varepsilon^{ijk}\sigma^j\sigma_\alpha\epsilon^k_- + g\sigma_\alpha\epsilon^i_- \end{aligned} \tag{A.23}$$

where α is a curved index on the S^3,

$$e^{w(\rho)} = \sinh\rho\,, \qquad f(\rho) = \frac{f_B\sinh^2\rho/2}{\cosh^2\rho/2 - f_B}\,, \qquad A^i_\alpha = f(\rho)w^i_\alpha$$

and $D^{ij}_\alpha \equiv (\partial_\alpha - \frac{i}{2}\sigma_\alpha)\delta^{ij} - 4ig\epsilon^{ikj}A^k_\alpha$.

We multiply each of the last two equations in (A.23) by σ^i, summing over i. Using (A.22) we find the derivative terms vanish. Multiplying by w_m^α and summing over α we are left with the following set of algebraic equations:

$$\begin{aligned} \left[-\frac{1}{2}\delta^{im} + i\varepsilon^{imk}\left(-\frac{1}{2}+4igf\right)\sigma^k\right]\epsilon_-^i + 2ig\left[-\frac{1}{2}\delta^{im} - \frac{i}{2}\varepsilon^{imk}\sigma^k\right]\epsilon_+^i &= 0 \\ \left[-\frac{1}{2}\delta^{im} + \varepsilon^{imk}\left(-\frac{1}{2}+4igf\right)\right]\epsilon_+^i - k2ig\left[-\frac{1}{2}\delta^{im} - \frac{i}{2}\varepsilon^{imk}\sigma^k\right]\epsilon_-^i &= 0 \end{aligned} \tag{A.24}$$

where $k \equiv 1 + \frac{8}{g}e^{-2w}f(1-f)$.

These are 6 homogenous equations for 6 spinors $\epsilon_\pm^i$. It is easy to verify that the determinant of coefficients is nonzero, and thus there is no nonzero solution to this system.

we conclude that there are also no ϵ type susys and therefore this background is not supersymmetric.

A.6 The conformal field theory

The dual field theory is the field theory on a stack of N coincident $M2$ branes with an external gauge field coupled to the R-symmetry currents. This gauge field is given by the boundary value of the bulk gauge fields. Though it is not possible to write the lagrangian for the interacting theory, it is possible to write a lagrangian for a single $M2$ brane. A single M2 branes is described by a supersymmetric theory with 8 free real scalar fields ϕ^a transforming as the vector of SO(8) and 8 free real fermions ψ^α transforming as the antichiral spinor representation, $\mathbf{8_s}$ of SO(8). The flat space lagrangian is[24]

$$S_{M2} = \int i\bar{\psi}^\alpha \partial\!\!\!/ \psi_\alpha + (\partial\phi^a)^2 \,. \tag{A.25}$$

The susy transformations for this theory are parameterized by a spinor in the chiral spinor representation, $\mathbf{8}_c$ of SO(8). they are:

$$\begin{aligned} \delta\phi^a &= \bar{\epsilon}\Gamma^a\psi \\ \delta\psi &= i\partial\!\!\!/\phi^a\Gamma^a\epsilon \,. \end{aligned} \tag{A.26}$$

Now we need to put this on a 3-sphere and add the gauge field.

Putting the theory on the sphere makes the derivatives become covariant derivatives with the spin-connection on the sphere. In addition, to preserve conformal symmetry we need to add an $\phi^2 R$ term to the lagrangian. This also adds another term in the supersymmetry transformation laws. One finds

$$S = \int d\Omega_3 \left[D_\mu\phi D^\mu\phi + i\bar{\psi}D\!\!\!\!/\psi + \frac{3}{4}\phi^2\right] \tag{A.27}$$

where we have set the radius of the S^3 to one. The supersymmetry transformations are

$$\begin{aligned} \delta_\epsilon\phi &= \bar{\epsilon}\psi \\ \delta_\epsilon\psi &= D\!\!\!\!/(\phi\epsilon) + i\phi\epsilon \end{aligned} \tag{A.28}$$

[24]Note that in lorentzian signature the spinors ψ are real, so $\bar{\psi} = \psi^t C$. In euclidean space we define $\bar{\psi} = \psi^t C$, where C is such that $\gamma_i^t = C\gamma^i C^{-1}$ and $C^t = -C$.

where ϵ obeys $D_\mu\epsilon = -\frac{i}{2}\gamma_\mu\epsilon$. Now in order to introduce the gauge field all we need to do is to add the gauge fields to the covariant derivatives in (A.27) (A.28), $D_\mu \to D_\mu + A_\mu$ where A_μ is the boundary value of the SU(2) gauge field we considered.

A.7 Details on the five dimensional solution

We start with the gauge fields in (5.3) and (5.4). We then compute

$$F^{IJ} = dA^{IJ} + [A,A]^{IJ} = iF^a L^{a\ IJ} + i\tilde{F}^a \tilde{L}^{a\ IJ}\,. \tag{A.29}$$

We chose the gauge fields so that $*F^a = F^a$ and $*\tilde{F}^a = -\tilde{F}^a$. We take both instantons to be SO(5) spherically symmetric. Using that $F^a_{\mu\nu}F^{b\ \mu\nu} = \tilde{F}^a_{\mu\nu}\tilde{F}^{b\ \mu\nu} = 4\delta^{ab}$ and $F^a_{\mu\nu}\tilde{F}^{b\ \mu\nu} = 0$ we can compute

$$F^{IJ}_{\mu\nu}F^{KJ\,\mu\nu} = -\left(L^{a\ IJ}L^{a\ KJ} + \tilde{L}^{a\ IJ}\tilde{L}^{a\ KJ}\right)4 = 8\delta^{IK} \tag{A.30}$$

where we used that $J^{a\ KJ} = -J^{a\ JK}$ and that $(J^aJ^a)^{IK} = 2\delta^{IK}$, $J^a = L^a, \tilde{L}^a$. Note that this implies that $\frac{1}{4}F^{IJ}F^{IJ} = 12$, which in turn gives $\beta = 1$ in (5.8), after using the action in [41]. We now need to observe that

$$\begin{aligned} F^a \wedge F^b &= -\tilde{F}^a \wedge \tilde{F}^b \sim \delta^{ab}\epsilon_4 \\ F^a \wedge \tilde{F}^b &= 0\,. \end{aligned} \tag{A.31}$$

This implies that

$$\epsilon_{IJKLMN}F^{KL}\wedge F^{MN} \sim \epsilon_4\left(L^{a\,KL}L^{a\,MN} - \tilde{L}^{a\,KL}\tilde{L}^{b\,MN}\right) = 0\,. \tag{A.32}$$

This implies that we are obeying the Chern Simons equations.

A.8 Search for negative modes from gauge fields

The background fields are given by

$$\begin{aligned} A^{ab} &= f\epsilon^{abc}w^c\,, & A^{AB} &= \tilde{f}\epsilon^{ABc}w^c \\ f &= \cos^2\frac{\theta}{2}\,, & \tilde{f} &= 1-f = \sin^2\frac{\theta}{2}\,. \end{aligned} \tag{A.33}$$

We can now compute

$$F^{ab} = f'd\theta\epsilon^{abc}w^c + f(1-f)w^aw^b\,. \tag{A.34}$$

We now consider the small fluctuation

$$-A^{Ba} = A^{aB} = h\epsilon^{aBc}w^c + g\delta^{aB}d\theta \tag{A.35}$$

where h, g are functions of θ. This leads to the following additional terms in F

$$\begin{aligned} \delta F^{aB} &= [h' + g(\tilde{f}-f)]d\theta\epsilon^{aBc}w^c + h(1-f-\tilde{f})w^aw^B \\ \delta F^{ab} &= -h^2w^aw^b + 2hg\epsilon^{abc}d\theta w^c \\ \delta F^{AB} &= +h^2w^Aw^B - 2hg\epsilon^{ABc}d\theta w^c\,. \end{aligned} \tag{A.36}$$

We now compute the action. In order to do this we need to remember that the unit normalized vielbein on S^3 is given by $e^a = w^a/2$.

JHEP02(2004)053

We can now compute F^2 for the above configuration (A.34) (A.35)

$$\begin{aligned}S &= \int_{S^4} F^{IJ}_{\alpha\beta} F^{IJ\,\alpha\beta} \\ &= 48\Omega_3 \int d\theta \sin^3\theta \Bigg[\frac{(f' + 2hg)^2 + (\tilde{f}' - 2hg)^2 + 2(h' + g(\tilde{f} - f))^2}{\sin^2\theta} + \\ &\qquad + 4\frac{[-f(1-f) + h^2]^2 + [-\tilde{f}(1-\tilde{f}) + h^2]^2 + 2(1 - f - \tilde{f})^2 h^2}{\sin^4\theta} \Bigg] .\end{aligned} \tag{A.37}$$

We see that the above expression vanishes for

$$g = \frac{1}{2}, \qquad h = \frac{1}{2}\sin\theta . \tag{A.38}$$

In fact (A.38) correspond to the values we would obtain if we start with the pure gauge configuration $A^{ab} = \epsilon^{abc} w^c$, $A^{AB} = 0$, $A^{aB} = 0$ and we do a gauge transformation by the gauge group element $e^{i\frac{\theta}{2}\Sigma}$ where Σ is the U(1) that exchanges $1, 2, 3$ with $4, 5, 6$.

We see that by turning on g and h continuously we can start from the instanton/anti-instanton configuration and go to the pure gauge configuration (A.38). Since varying h and g we can find a path that gets rid of the original gauge field configuration, we expect to get a negative mode for small fluctuations of h and g. In principle we need to consider the most general fluctuation of the gauge fields in order to find all negative modes. Since the fluctuations parametrized by h, g are general enough to provide a path in field space that gets rid of the instanton/anti-instanton configuration it is very likely that the negative mode we find by considering small fluctuations of h, g is the most negative mode.

Denoting the small fluctuations of the gauge field A around the background (A.34) by δA we can expand the Yang-Mills action to second order. This will schematically lead to an expression of the form $\int \delta A \mathcal{O} \delta A$, where $\mathcal{O}$ is some operator. We are interested in finding eigenvalues for this operators $\mathcal{O}\delta A = \lambda \delta A$. In order to do this we consider the auxiliary action given by

$$S^{(2)}_{\rm aux} = \int \delta A \mathcal{O} \delta A - \lambda (\delta A)^2 \tag{A.39}$$

and we look for solutions of this action that are non-singular. These will exist only for special values of λ. If $\lambda \neq 0$ we do not have to worry about gauge fixing. We can compute the first term in (A.39) by expanding (A.37) to second order in h, g. We find

$$S^{(2)}_{\rm aux} = 48\Omega_3 \int d\theta \sin\theta \left[2(h' - g\cos\theta)^2 - 4h^2 - 4hg\sin\theta - \frac{\lambda}{4}(\sin^2\theta g^2 + 8h^2) \right] . \tag{A.40}$$

In order to find the eigenvalue λ we just solve the equations of motion for S. The equation of motion for g is algebraic, so we replace the resulting value back into the action and we get the following action for h

$$S^{(2)}_{\rm aux} = 48\Omega_3 \int d\theta \sin\theta \left[2h'^2 - 4h^2 - 2\lambda h^2 - 4\frac{(h'\cos\theta + h\sin\theta)^2}{2\cos^2\theta - \frac{\lambda}{4}\sin^2\theta} \right] . \tag{A.41}$$

The equations of motion for h read

$$\begin{aligned}&-2(\sin\theta h')' - (4+2\lambda)\sin\theta h + \\ +4&\left(\frac{\sin\theta\cos\theta(\cos\theta h' + \sin\theta h)}{2\cos^2\theta - \frac{\lambda}{4}\sin^2\theta}\right)' - 4\frac{\sin^2\theta(h'\cos\theta + \sin\theta h)}{2\cos^2\theta - \frac{\lambda}{4}\sin^2\theta} = 0\end{aligned} \tag{A.42}$$

Examining the equation near $\theta \sim 0$ we find that the regular solution goes as θ^2. We find a similar behavior at $\theta = \pi$. The value of λ should be chosen so that (A.42) has a nontrivial solution which is regular at $\theta = 0, \pi$. We find a regular solution for $\lambda = -4$ which is $h = \sin^2\theta$. We see that this is the lowest eigenvalue. We can check numerically that lower values of λ lead to solutions for h which do not cross the real axis if they are regular at one of the two ends.

Now we need to consider the radial dependence. We choose a gauge $A_\rho = 0$. The only terms with ρ derivatives will come from $F_{\rho\alpha} = \partial_\rho A_\alpha$, where α is an index on S^4. So we see that for a given eigenvector with eigenvalue λ the lagrangian will be of the form

$$\begin{aligned} S^{(2)} &\sim \int d\rho \left[2e^{2\omega} F^{IJ}_{\rho\alpha} F^{IJ}_{\rho\alpha} + \lambda A^{IJ}_\alpha A^{IJ}_\alpha\right] \\ S^{(2)} &\sim \int d\rho \left[2e^{2\omega}(\phi')^2 + \lambda\phi^2\right] \end{aligned} \tag{A.43}$$

where $e^{2\omega}$ was given in (5.8) and ϕ is a field which encodes the ρ dependence of the eigenvector with eigenvalue λ. In other words, we consider a gauge field fluctuation $\delta A = \phi(\rho)\delta A^{(\lambda)}$, where $\delta A^{(\lambda)}$ is the eigenvector from (A.39).

We are now interested in understanding if the operator that appears in the last line of (A.43) has negative modes. We impose the boundary condition $\phi = 0$ at $\rho = \pm\infty$. We now write down the equation of motion for (A.43)

$$\partial_\rho(2e^{2\omega}\partial_\rho\phi) - \lambda\phi = 0\,. \tag{A.44}$$

The operator in (A.43) will have a negative mode if the solution to (A.44) with boundary conditions $\phi(\rho = 0) = 1$ and $\phi'(\rho = 0) = 0$ changes sign as $\rho \to \infty$. Setting $\lambda = -4$ and analyzing the equation numerically we can see that it does not change sign.[25]

The final conclusion is that there is no negative mode of the type we looked for. There might be a negative mode that has a more complicated wavefunction on S^4.

A.9 Analysis of the scalar fields in the instanton background

Using (5.3) and (5.4) we find that

$$F^{IJ}_{\mu\nu} F^{KL\,\mu\nu} = -4\left(f L^{a\ IJ} L^{a\ KL} + \tilde{L}^{a\ IJ}\tilde{L}^{a\ KL}\right). \tag{A.45}$$

We can now use that if I, J, K, L are all $1, 2, 3$ then we have[26]

$$L^{a\ IJ} L^{a\ KL} = -\left(\delta^{IK}\delta^{JL} - \delta^{IL}\delta^{JK}\right) \tag{A.46}$$

[25] We can have negative modes if $\lambda < -5.67 \pm 0.03$.

[26] Note that $L^{a\,IJ} = -\epsilon^{aIJ}$.

We now turn to a discussion of the scalar fields. They are given in terms of a matrix T_{IJ} which has determinant one. This implies that we can write

$$T^{-1}_{IJ} = \delta_{IJ} + \phi_{IJ} + h\delta_{IJ}\,, \qquad 12h = \phi_{IJ}\phi_{IJ} + \cdots \tag{A.47}$$

This ensures that T has determinant one to second order.

We now evaluate the kinetic term and potential using the expressions in [41]. We find

$$S = \int \frac{1}{4}\partial\phi_{IJ}\partial\phi_{IJ} - g^2\phi_{IJ}\phi_{IJ}\,. \tag{A.48}$$

To evaluate this we need to compute T_{IJ} from (A.47), which is $T_{IJ} = (1+h)\delta_{IJ} - \phi_{IJ} + \cdots$, where the dots include a quadratic term in ϕ which is traceless and does not contribute to the computation leading to (A.48). After setting $g = 1$ we see that we get the correct mass, $m^2 = -4$. We are working in units where the radius of AdS is one.

We need to add the term that comes from the coupling of the scalars to the gauge fields given by

$$\frac{1}{4}T^{-1}_{IK}T^{-1}_{JL}F^{IJ}F^{KL}\,. \tag{A.49}$$

The scalars can be decomposed under SO(6) $\to$ SO(3) $\times$ SO(3) as $20 \to (5,1) + (1,5) + (3,3) + (1,1)$. Looking at (A.47) we see that there are two kinds of couplings that can appear from (A.49), either one ϕ from each from each T in (A.49) or a coupling to h in (A.47) from one of the Ts in (A.49). The coupling from h does not depend on the type of ϕ we consider and is equal to

$$2h\frac{1}{4}F^{IJ}F^{IJ} = 2h12 = 2\phi_{IJ}\phi_{IJ}\,. \tag{A.50}$$

The two ϕ coupling will depend on the type of scalar we consider. For the $(3,3)$ scalars there is no extra coupling and (A.50) is the full answer. For the (5,1) scalars we can now use (A.45) and (A.46) to find the extra term

$$-\phi_{IJ}\phi_{IJ}\,, \qquad I,J = 1,2,3\,. \tag{A.51}$$

We can do a similar analysis for the $(1,1)$ field, which gives us the extra term

$$2\phi_{IJ}\phi_{IJ} \tag{A.52}$$

where we used that ϕ is diagonal and rewrote it as in (A.52).

Now we study the question of whether there are negative modes. We need to consider the equation

$$-e^{-4\omega}\partial_\rho(e^{4\omega}\partial_\rho\phi) + (m^2 + \gamma e^{-4\omega})\phi = \lambda\phi \tag{A.53}$$

where $m^2 = -4$ and ω is given in (5.9). The parameter γ is a number that depends on the type of scalar field. For the scalars in the (3,3),(1,1),(1,5),(5,1) representations $\gamma = 8, 16, 4, 4$ respectively. It is important to note that $\gamma > 0$ for all the scalar fields considered.

Solving the equation (A.53) with the initial condition $\phi(0) = 1$, $\phi'(0) = 0$ we see that if $\gamma > 0$ then the solution stays positive. This means that the corresponding operator does not have any negative modes.

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[3] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[4] G.J. Galloway, K. Schleich, D. Witt and E. Woolgar, *The AdS/CFT correspondence conjecture and topological censorship*, *Phys. Lett.* **B 505** (2001) 255 [hep-th/9912119].

[5] E. Witten and S.T. Yau, *Connectedness of the boundary in the AdS/CFT correspondence*, *Adv. Theor. Math. Phys.* **3** (1999) 1635 [hep-th/9910245].

[6] M.-L. Cai and G.J. Galloway, *Boundaries of zero scalar curvature in the AdS/CFT correspondence*, *Adv. Theor. Math. Phys.* **3** (1999) 1769 [hep-th/0003046].

[7] S.B. Giddings and A. Strominger, *Axion induced topology change in quantum gravity and string theory*, *Nucl. Phys.* **B 306** (1988) 890.

[8] S.B. Giddings and A. Strominger, *String wormholes*, *Phys. Lett.* **B 230** (1989) 46;
S. Pratik Khastgir and J. Maharana, *Topology changing processes and symmetries of string effective action*, *Nucl. Phys.* **B 406** (1993) 145 [hep-th/9302069];
G.W. Gibbons, M.B. Green and M.J. Perry, *Instantons and seven-branes in type-IIB superstring theory*, *Phys. Lett.* **B 370** (1996) 37 [hep-th/9511080];
J.Y. Kim, H.W. Lee and Y.S. Myung, *D-instanton and d-wormhole*, *Phys. Lett.* **B 400** (1997) 32 [hep-th/9612249];
J.Y. Kim, H.W. Lee and Y.S. Myung, *Negative modes in the four-dimensional stringy wormholes*, *Phys. Rev.* **D 56** (1997) 6684 [hep-th/9701116];
M. Gutperle and M. Spalinski, *Supergravity instantons and the universal hypermultiplet*, *J. High Energy Phys.* **06** (2000) 037 [hep-th/0005068].

[9] M. Cadoni and M. Cavaglia, *Cosmological and wormhole solutions in low-energy effective string theory*, *Phys. Rev.* **D 50** (1994) 6435 [hep-th/9406053];
V.D. Dzhunushaliev, *5d Kaluza-Klein's wormhole between two event horizons*, gr-qc/9603007;
H. Kim, *Axionic wormholes: more on their classical and quantum aspects*, *Nucl. Phys.* **B 527** (1998) 311 [hep-th/9706015];
V.D. Dzhunushaliev and H.J. Schmidt, *Wormholes and flux tubes in the 7d gravity on the principal bundle with* SU(2) *gauge group as the extra dimensions*, *Phys. Rev.* **D 62** (2000) 044035 [gr-qc/9911080];
S.A. Hayward, S.-W. Kim and H.-j. Lee, *Dilatonic wormholes: construction, operation, maintenance and collapse to black holes*, *Phys. Rev.* **D 65** (2002) 064003 [gr-qc/0110080];
S.I. Vacaru and D. Singleton, *Ellipsoidal, cylindrical, bipolar and toroidal wormholes in 5d gravity, J. Math. Phys.* **43** (2002) 2486 [hep-th/0110272];
M. Gutperle and W. Sabra, *Instantons and wormholes in Minkowski and (A)dS spaces*, *Nucl. Phys.* **B 647** (2002) 344 [hep-th/0206153];
K.A. Bronnikov and S.-W. Kim, *Possible wormholes in a brane world*, *Phys. Rev.* **D 67** (2003) 064027 [gr-qc/0212112].

JHEP02(2004)053

[10] S.R. Coleman and K.-M. Lee, *Wormholes made without massless matter fields*, *Nucl. Phys.* **B 329** (1990) 387.

[11] S.R. Coleman and K.-M. Lee, *Big wormholes and little interactions*, *Nucl. Phys.* **B 341** (1990) 101.

[12] A. Hosoya and W. Ogura, *Wormhole instanton solution in the Einstein Yang-Mills system*, *Phys. Lett.* **B 225** (1989) 117.

[13] E.E. Donets and D.V. Galtsov, *Continuous family of Einstein Yang-Mills wormholes*, *Phys. Lett.* **B 294** (1992) 44 [gr-qc/9209008].

[14] S.-J. Rey, *The collective dynamics and the correlations of wormholes in quantum gravity*, *Nucl. Phys.* **B 319** (1989) 765; *The axion dynamics in wormhole background*, *Phys. Rev.* **D 39** (1989) 3185.

[15] N. Seiberg and E. Witten, *The D1/D5 system and singular CFT*, *J. High Energy Phys.* **04** (1999) 017 [hep-th/9903224].

[16] G.W. Gibbons, S.W. Hawking and M.J. Perry, *Path integrals and the indefiniteness of the gravitational action*, *Nucl. Phys.* **B 138** (1978) 141.

[17] J.P. Gauntlett, J.B. Gutowski, C.M. Hull, S. Pakis and H.S. Reall, *All supersymmetric solutions of minimal supergravity in five dimensions*, *Class. and Quant. Grav.* **20** (2003) 4587 [hep-th/0209114];
J.P. Gauntlett and S. Pakis, *The geometry of $D = 11$ Killing spinors*, *J. High Energy Phys.* **04** (2003) 039 [hep-th/0212008];
J.P. Gauntlett and J.B. Gutowski, *All supersymmetric solutions of minimal gauged supergravity in five dimensions*, *Phys. Rev.* **D 68** (2003) 105009 [hep-th/0304064];
J.B. Gutowski, D. Martelli and H.S. Reall, *All supersymmetric solutions of minimal supergravity in six dimensions*, *Class. and Quant. Grav.* **20** (2003) 5049 [hep-th/0306235];
M.M. Caldarelli and D. Klemm, *All supersymmetric solutions of $N = 2$, $D = 4$ gauged supergravity*, *J. High Energy Phys.* **09** (2003) 019 [hep-th/0307022]; *Supersymmetric goedel-type universe in four dimensions*, *Class. and Quant. Grav.* **21** (2004) L17 [hep-th/0310081].

[18] E. Witten, private communication.

[19] A. Karch and L. Randall, *Open and closed string interpretation of SUSY CFT's on branes with boundaries*, *J. High Energy Phys.* **06** (2001) 063 [hep-th/0105132];
M. Porrati, *Higgs phenomenon for the graviton in AdS space*, *Mod. Phys. Lett.* **A 18** (2003) 1793 [hep-th/0306253];
M. Porrati, *Higgs phenomenon for 4-d gravity in anti de Sitter space*, *J. High Energy Phys.* **04** (2002) 058 [hep-th/0112166].

[20] G.T. Horowitz and D. Marolf, *A new approach to string cosmology*, *J. High Energy Phys.* **07** (1998) 014 [hep-th/9805207].

[21] I.R. Klebanov and E. Witten, *AdS/CFT correspondence and symmetry breaking*, *Nucl. Phys.* **B 556** (1999) 89 [hep-th/9905104].

[22] R.R. Mazzeo, *Unique continuation of infinity and embedded eigenvalues for asymptotically hyperbolic manifolds*, *Am. J. Math.* **113** (1991) 25;
R. Schoen and S.T. Yau, *Conformally flat manifolds, kleinian groups and scalar curvature*, *J. Diff. Geom.* **20** (1984) 479;

J.M. Lee, *The spectrum of an asymptotically hyperbolic Einstein manifold*, *Comm. Anal. Geom.* **3** (1995), no. 1-2, 253 [dg-ga/9409003].

[23] F. Larsen and E.J. Martinec, *Currents and moduli in the (4,0) theory*, *J. High Energy Phys.* **11** (1999) 002 [hep-th/9909088].

[24] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *The supergravity dual of $N = 1$ super Yang-Mills theory*, *Nucl. Phys.* **B 569** (2000) 451 [hep-th/9909047].

[25] W.M. Fairbairn, T. Fulton and W. Klink, *Finite and disconnected subgroups of* SU_3 *and their application to the elementary particle physics spectrum*, *J. Math. Phys.* **5** (1964) 1038.

[26] A. Hanany and Y.-H. He, *A monograph on the classification of the discrete subgroups of* SU(4), *J. High Energy Phys.* **02** (2001) 027 [hep-th/9905212].

[27] D. Bak, M. Gutperle and S. Hirano, *A dilatonic deformation of AdS_5 and its field theory dual*, *J. High Energy Phys.* **05** (2003) 072 [hep-th/0304129].

[28] B.T. McInnes, *Topologically induced instability in string theory*, *J. High Energy Phys.* **03** (2001) 031 [hep-th/0101136].

[29] J. Polchinski and M.J. Strassler, *The string dual of a confining four-dimensional gauge theory*, hep-th/0003136.

[30] D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Correlation functions in the CFT_d/AdS_{d+1} correspondence*, *Nucl. Phys.* **B 546** (1999) 96 [hep-th/9804058].

[31] S. de Haro, S.N. Solodukhin and K. Skenderis, *Holographic reconstruction of spacetime and renormalization in the AdS/CFT correspondence*, *Commun. Math. Phys.* **217** (2001) 595 [hep-th/0002230];
M. Bianchi, D.Z. Freedman and K. Skenderis, *Holographic renormalization*, *Nucl. Phys.* **B 631** (2002) 159 [hep-th/0112119];
K. Skenderis, *Lecture notes on holographic renormalization*, *Class. and Quant. Grav.* **19** (2002) 5849 [hep-th/0209067].

[32] L. Bers, *Simultaneous uniformization*, *Bull. Amer. Math. Soc.* **66** (1960) 94; *Spaces of kleinian groups*, in *Maryland conference in several complex variables I*, p. 9–34, *Lect. Notes in Math.* **155**, Springer-Verlag, Berlin 1970.

[33] K. Krasnov, *Holography and Riemann surfaces*, *Adv. Theor. Math. Phys.* **4** (2000) 929 [hep-th/0005106].

[34] W.P. Thurston, *The geometry and topology of three-manifolds*, Lecture Notes, Princeton University, 1980, http://www.msri.org/publications/books/gt3m/.

[35] S. Aminneborg, I. Bengtsson, D. Brill, S. Holst and P. Peldan, *Black holes and wormholes in 2+1 dimensions*, *Class. and Quant. Grav.* **15** (1998) 627 [gr-qc/9707036];
S. Aminneborg, I. Bengtsson, S. Holst and P. Peldan, *Making anti-de Sitter black holes*, *Class. and Quant. Grav.* **13** (1996) 2707 [gr-qc/9604005];
M. Bañados, *Constant curvature black holes*, *Phys. Rev.* **D 57** (1998) 1068 [gr-qc/9703040];
K. Krasnov, *Analytic continuation for asymptotically AdS 3d gravity*, *Class. and Quant. Grav.* **19** (2002) 2399 [gr-qc/0111049].

[36] S.-J. Rey, *Space-time wormholes with Yang-Mills fields*, *Nucl. Phys.* **B 336** (1990) 146; *The confining phase of superstrings and axionic strings*, *Phys. Rev.* **D 43** (1991) 526.

JHEP02(2004)053

[37] A.K. Gupta, J. Hughes, J. Preskill and M.B. Wise, *Magnetic wormholes and topological symmetry*, *Nucl. Phys.* **B 333** (1990) 195.

[38] B. de Wit and H. Nicolai, *The consistency of the S^7 truncation in $D = 11$ supergravity*, *Nucl. Phys.* **B 281** (1987) 211.

[39] L.J. Romans, *The* SO(5) *sectors of gauged $N = 8$ supergravity*, *Phys. Lett.* **B 131** (1983) 83.

[40] M. Cvetič, H. Lu and C.N. Pope, *Four-dimensional $N = 4$,* SO(4) *gauged supergravity from $D = 11$*, *Nucl. Phys.* **B 574** (2000) 761 [hep-th/9910252].

[41] M. Cvetič, H. Lu, C.N. Pope, A. Sadrzadeh and T.A. Tran, *Consistent* SO(6) *reduction of type-IIB supergravity on S^5*, *Nucl. Phys.* **B 586** (2000) 275 [hep-th/0003103].

[42] A. Strominger, *Vacuum topology and incoherence in quantum gravity*, *Phys. Rev. Lett.* **52** (1984) 1733.

[43] G.V. Lavrelashvili, V.A. Rubakov and P.G. Tinyakov, *Disruption of quantum coherence upon a change in spatial topology in quantum gravity*, *Sov. Phys. JETP Lett.* **46** (1987) 167 [*Zh. Eksp. Teor. Fiz.* **46** (1987) 134]; *Particle creation and destruction of quantum coherence by topological change*, *Nucl. Phys.* **B 299** (1988) 757.

[44] S.W. Hawking, *Quantum coherence down the wormhole*, *Phys. Lett.* **B 195** (1987) 337.

[45] S.R. Coleman, *Black holes as red herrings: topological fluctuations and the loss of quantum coherence*, *Nucl. Phys.* **B 307** (1988) 867.

[46] S.R. Coleman, *Why there is nothing rather than something: a theory of the cosmological constant*, *Nucl. Phys.* **B 310** (1988) 643;
S.R. Coleman and K.-M. Lee, *Escape from the menace of the giant wormholes*, *Phys. Lett.* **B 221** (1989) 242.

[47] S.-J. Rey, *Holographic principle and topology change in string theory*, *Class. and Quant. Grav.* **16** (1999) L37 [hep-th/9807241].

[48] L. Fidkowski, V. Hubeny, M. Kleban and S. Shenker, *The black hole singularity in AdS/CFT*, hep-th/0306170.

[49] E. Witten, *AdS/CFT correspondence and topological field theory*, *J. High Energy Phys.* **12** (1998) 012 [hep-th/9812012].

[50] J.M. Maldacena, G.W. Moore and N. Seiberg, *D-brane charges in five-brane backgrounds*, *J. High Energy Phys.* **10** (2001) 005 [hep-th/0108152].

[51] M. Henningson and K. Skenderis, *The holographic Weyl anomaly*, *J. High Energy Phys.* **07** (1998) 023 [hep-th/9806087].

[52] M.A. Awada, M.J. Duff and C.N. Pope, *$N = 8$ supergravity breaks down to $N = 1$*, *Phys. Rev. Lett.* **50** (1983) 294.

[53] A. Das, SO(4) *invariant extended supergravity*, *Phys. Rev.* **D 15** (1977) 2805.

[54] S.J. Gates Jr. and B. Zwiebach, *Gauged $N = 4$ supergravity theory with a new scalar potential*, *Phys. Lett.* **B 123** (1983) 200.

[55] P. van Nieuwenhuizen and A. Waldron, *On euclidean spinors and wick rotations*, *Phys. Lett.* **B 389** (1996) 29 [hep-th/9608174].

[56] D.Z. Freedman, SO(3) *invariant extended supergravity*, *Phys. Rev. Lett.* **38** (1977) 105.